Linear Modeling of the Adversarial Noise Space

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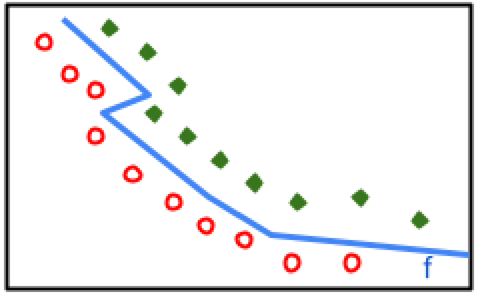
Adversarial Attacks

Among the various adversarial attacks, we restrict to perburbation-based attacks

Problem: Given a classifier C_f



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with associated classifier $C_f = \text{argmax}_{i \in \{1, \ldots, c\}} (f(\cdot))_i$

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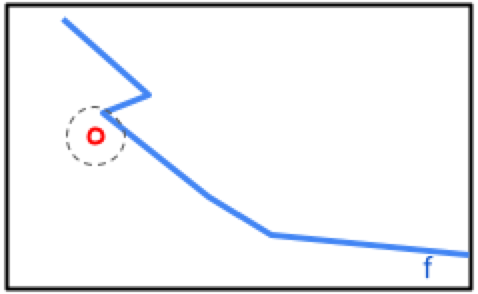

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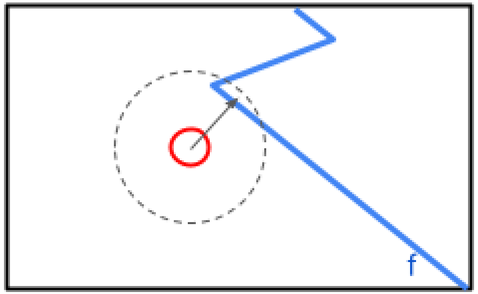

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Two Paradigms: Specific vs. Universal

Specific Attacks

For each $\mathbf{x}^{(i)}$, learn $\epsilon^{(i)}$ such that
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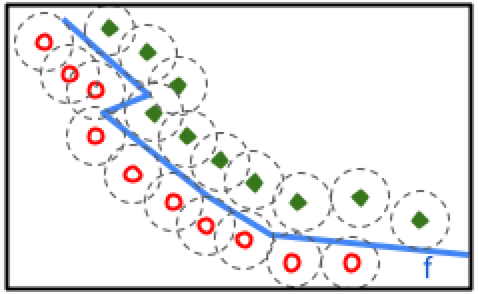

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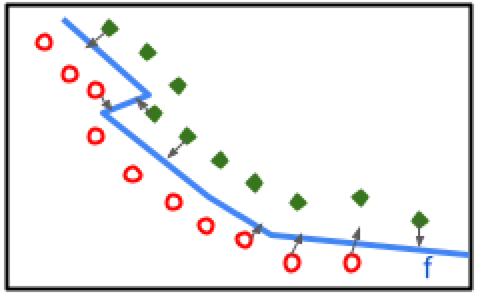

High fooling rate

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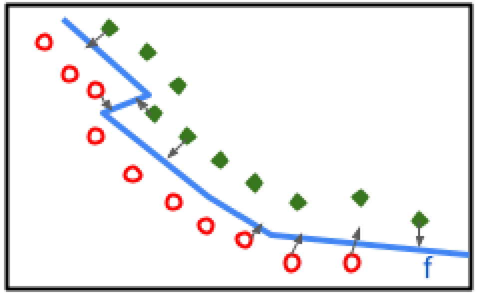

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Universal Attack

Learn ϵ such that, for each $\mathbf{x}^{(i)}$,

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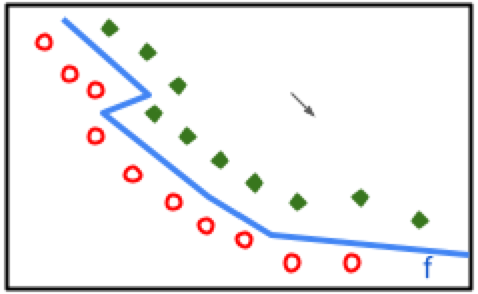

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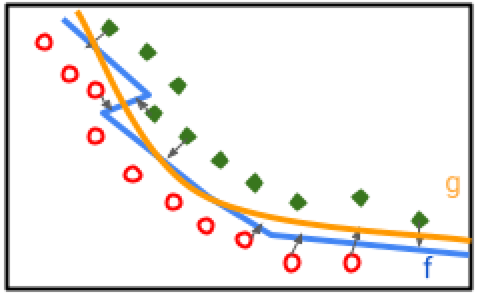

High fooling rate
Poor transferability

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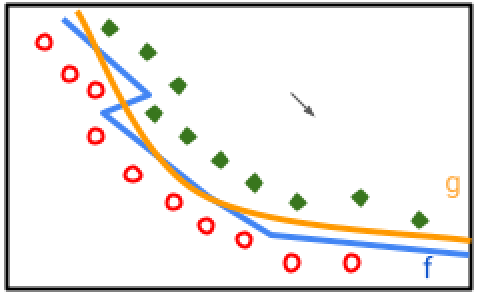

Poor fooling rate
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Proposed Attack

Principle

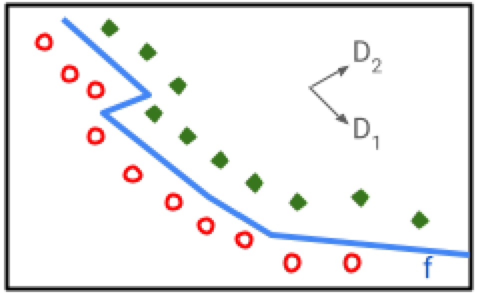
LIMANS

Linear Modeling of the Adversarial Noise Space

$$\mathbf{x}^{(i)'} = \mathbf{x}^{(i)} + D\mathbf{v}^{(i)}$$

$D = [D_1, \ldots, D_M]$ are universal directions (*size of* $\mathbf{x}^{(i)}$)

$\mathbf{v}^{(i)} = [\mathbf{v}_1^{(i)}, \ldots, \mathbf{v}_M^{(i)}]$ are specific coding vectors (*scalars*)



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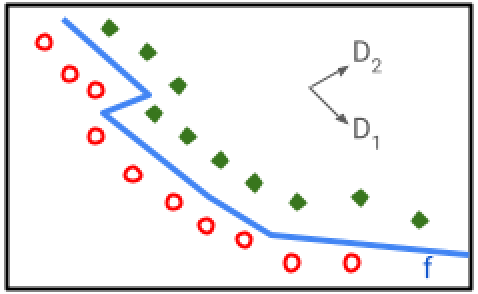

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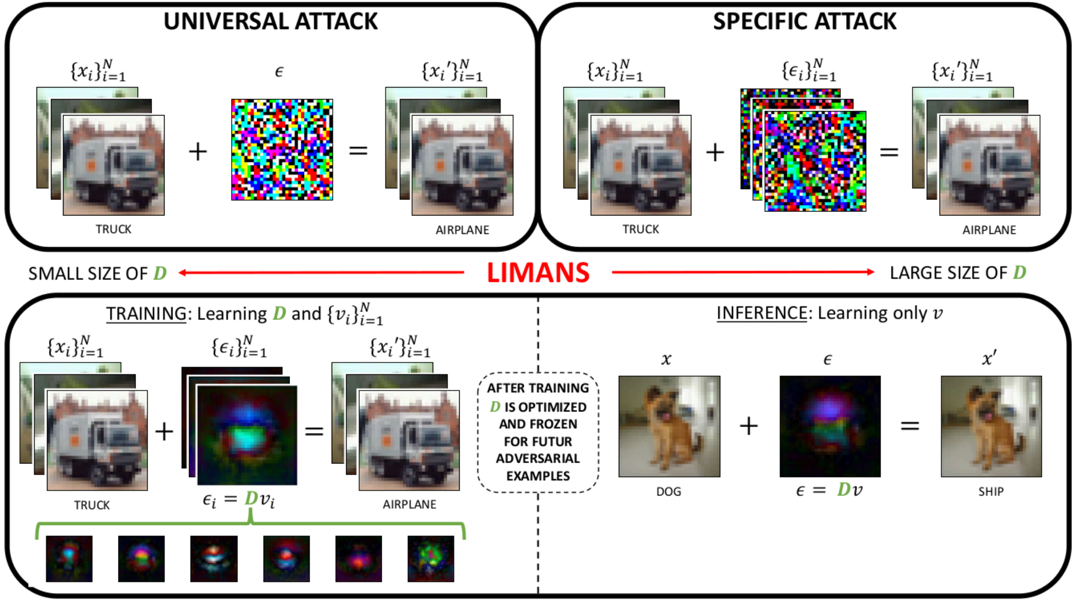

By tuning the size of D, LIMANS bridges the gap between universal and specific attacks

Optimization Problem

$$\underset{\substack{D=[D_1,\ldots,D_M]\in\mathbb{R}^{P\times M}\\ V=[\mathbf{v}^{(1)},\ldots\mathbf{v}^{(N)}]\in\mathbb{R}^{M\times N}}}{\text{maximize}} \frac{1}{N}\sum_{i=1}^{N} 1_{\{C_f(\mathbf{x}^{(i)\prime})\neq C_f(\mathbf{x}^{(i)})\}}$$

$$\text{s.t.} \quad \begin{cases} \mathbf{x}^{(i)\prime} = \mathbf{x}^{(i)} + D\mathbf{v}^{(i)} \in \mathcal{X} & ,(\forall i \in \{1,\ldots,N\}) \quad \textit{Valid examples} \\ \|D\mathbf{v}^{(i)}\|_p \leq \delta_p & ,(\forall i \in \{1,\ldots,N\}) \quad \textit{Small perturbations} \\ \|D_j\|_p = 1 & ,(\forall j \in \{1,\ldots,M\}) \quad \textit{Normalized directions} \end{cases}$$

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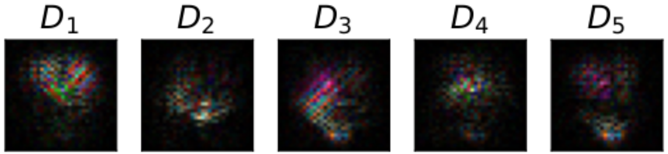
- The 3 constraints → we propose 2 different relaxations

Numerical Experiments

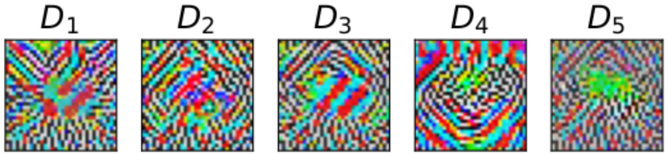
Visualisation of Adversarial Directions

Setting: Attack a VGG11 (top) or robust ResNet50 (bottom) on CIFAR10. Learn $M = 5$ directions.

ℓ_2-attack



ℓ_∞-attack



Having a linear model of the adversarial noise space allows for visual inspection of the adversarial directions, which is advantageous for understanding the attack behavior.

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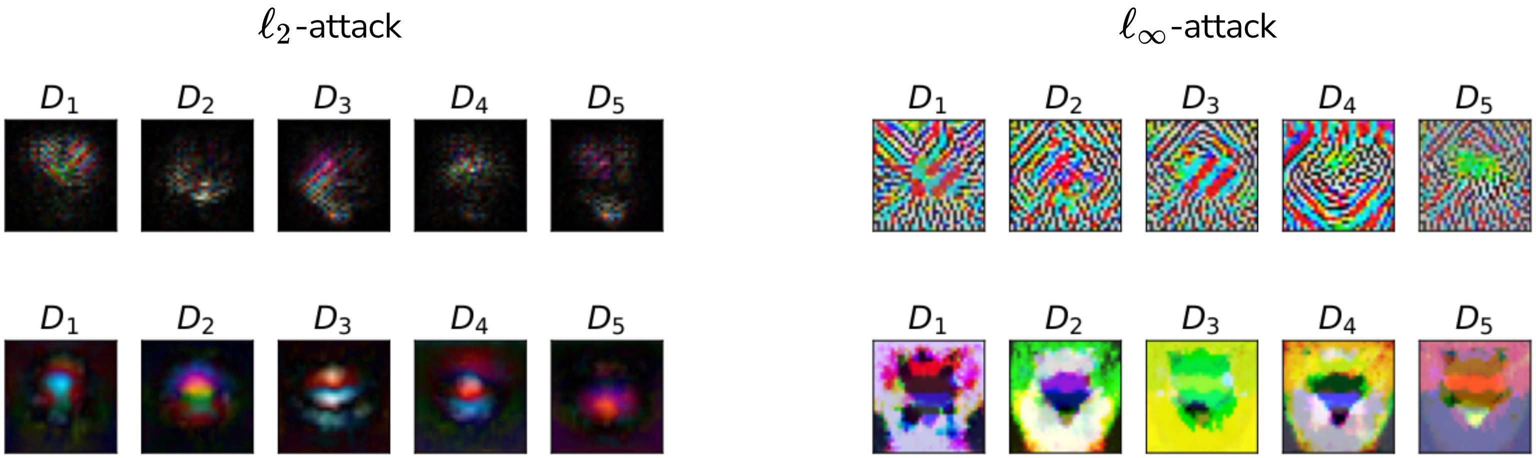

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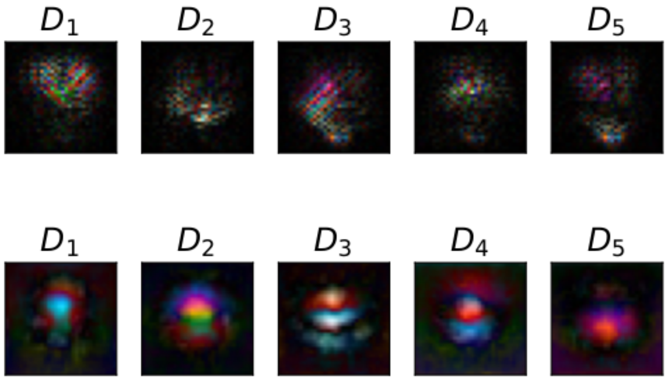
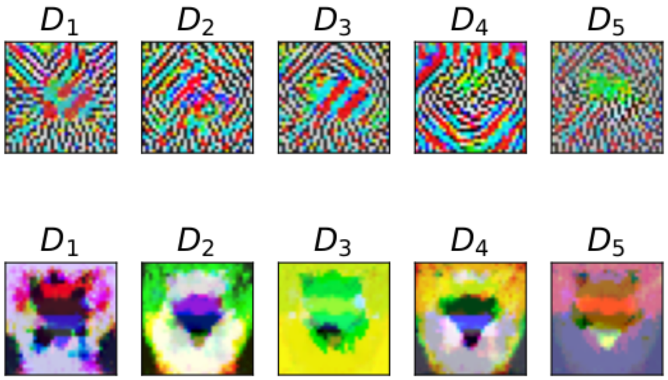


Mostly local spots

ℓ_∞-attack



Mostly edges and corners

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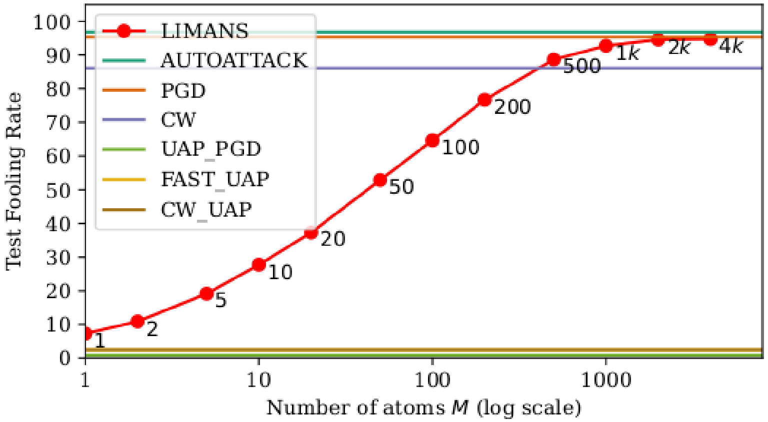
Impact of the Number of Directions

Setting: Attack a VGG11 on CIFAR10 with ℓ_2-attacks.



Specific: AutoAttack, PGD, CW

Universal: UAP PGD, FAST UAP, CW UAP

As M increases, LIMANS progressively narrows the performance gap with specific attacks

Transferability

Setting: Attack a VGG11 on CIFAR10. Evaluate fooling performance on target classifiers (columns).

	MobileNet	ResNet50	DenseNet	VGG	R-r18	R-wrn-34-10
AutoAttack	62.5	43.0	44.0	100	2.7	2.7
VNI-FGSM	69.3	62.6	61.4	96.5	3.0	2.6
NAA	42.3	14.5	1.8	71.6	1.6	1.2
RAP	46.5	39.5	40.9	73.8	3.3	3.4
Ours	97.4	87.5	81.5	91.0	11.5	12.6

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Our model yields better transferability performance, i.e. **source classifier** $\neq$ **target classifier**

Bypassing Attack Detectors

Setting: Attack a VGG11 on CIFAR10. Train systems to detect adversarial attacks (columns)

Classifiers / Detectors	detect FGSM	detect PGD	detect AutoAttack	detect LIMANS 10
FGSM	91.1	91.1	91.1	83.4
PGD	90.6	91.1	91.1	55.9
Autoattack	89.9	90.9	91.1	52.7
LIMANS_{10}	75.7	81.0	81.6	88.9
LIMANS_{500}	17.5	25.6	31.8	26.6
LIMANS_{1000}	15.9	26.1	32.1	**21.7**
LIMANS_{4000}	**15.6**	**23.7**	**28.2**	31.1

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(the lower, the better)

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Autoattack	89.9	90.9	91.1	52.7
$LIMANS_{10}$	75.7	81.0	81.6	88.9
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$LIMANS_{4000}$	15.6	23.7	28.2	31.1

RAUD (*Robust Accuracy Under Defense*): quantifies the percentage of successful attacks detected

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LIMANS attacks consistently evade detection

outperforming specific attacks even at M = 10 and exhibiting robustness from M ≥ 500

Conclusion

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Experimental findings:

- Bridge the gap between specific and universal attacks
- Allows visual inspection of the learned directions
- Show great transferability
- Bypass adversarial detectors

Thank you for your attention!

Questions?

Download the paper

Take-home message: Attacks are framed as specific linear combinations of universal adversarial directions